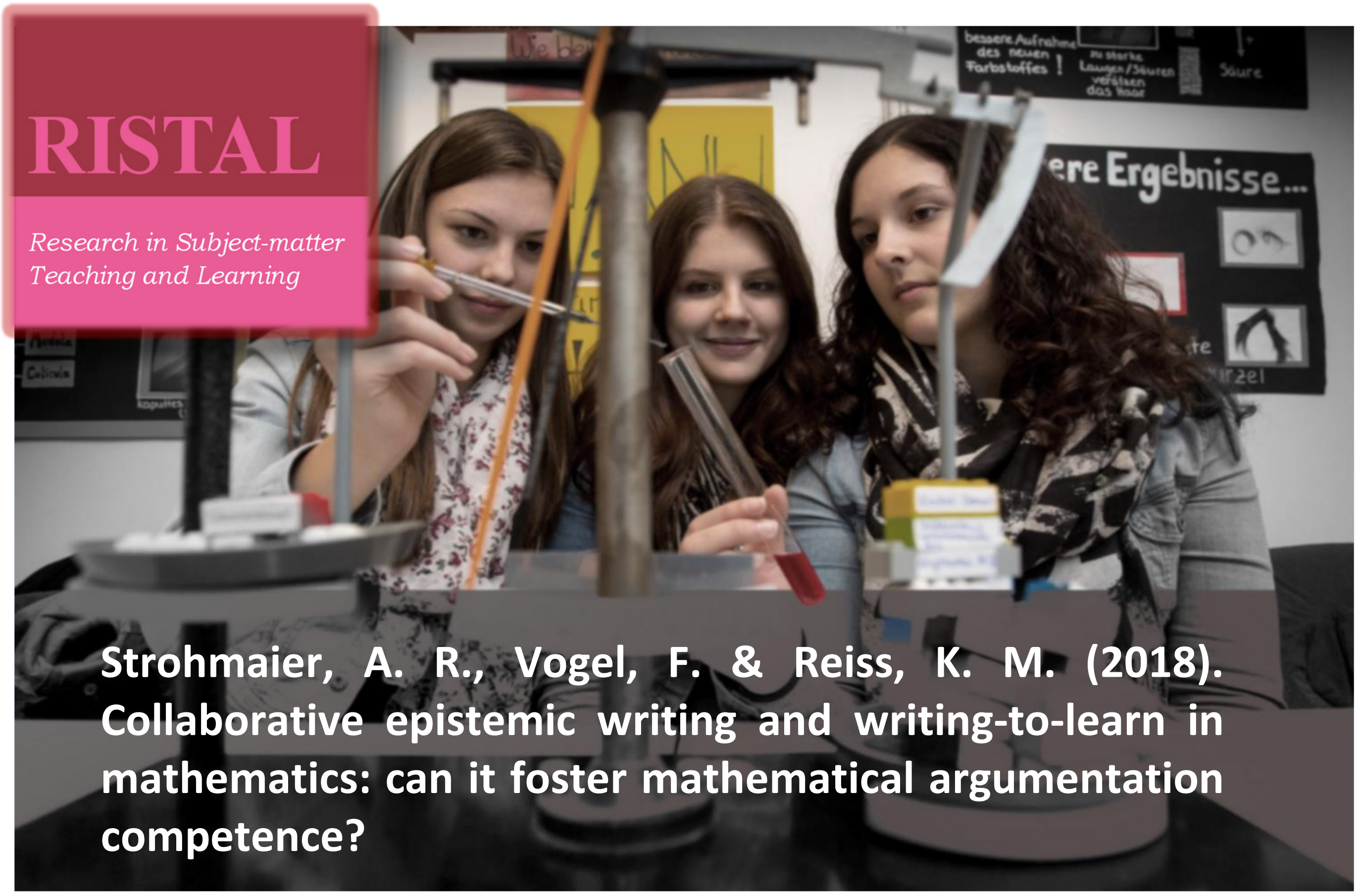

RISTAL 1 / 2018

Research in Subject-matter Teaching and Learning

Launch issue

Citation:
Strohmaier, A. R., Vogel, F. & Reiss, K. M. (2018). Collaborative epistemic writing and writing-to-learn in mathematics: can it foster mathematical argumentation competence? *RISTAL, 1*, 134–148.

DOI: https://doi.org/10.23770/rt1817

ISSN 2616-7697

RISTAL. Research in Subject-matter Teaching and Learning 1 (2018), pp. 134–148

Collaborative epistemic writing and writing-to-learn in mathematics: can it foster mathematical argumentation competence?

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Abstract

Writing is not only a tool for communication but can assist in generating, elaborating and structuring knowledge, which is commonly referred to as writing-to-learn. This can be beneficial in many mathematical learning processes. However, most research on mathematical writing has focused on individual writing-to-learn in problem solving activities. In contrast, only few studies considered possible benefits for the fostering of mathematical argumentation and proving, particularly when writing-to-learn occurs in collaboration. We review fundamental principles of writing-to-learn in mathematics education and elaborate how it could be implemented in collaborative learning settings. We refer to previous research on fostering mathematical argumentation skills as well as to theories of writing-to-learn from other areas within mathematics education. Finally, we provide guidelines for designing collaborative knowledge-constituting writing activities, discussing the importance of prompts and guidance.

Keywords

epistemic writing, writing-to-learn, mathematical writing, collaborative learning, mathematical argumentation, heuristic worked examples, collaboration scripts

"The three R's - Reading, Writing, and 'Rithmetic"
Sir William Curtis (1752-1829), toast at a school dinner

1 Introduction

The anecdotal notion of the *three R's* illustrates how language and mathematics arguably reflect the two most fundamental domains taught at school. Today, the three R's have long been replaced by more elaborated goals of education. For example, the *Programme for International Student Assessment* (PISA) assesses students' science, reading, and mathematical literacy as the three core indicators for successful educational systems (OECD, 2016). However, while reading and arithmetic are still essential to these recent approaches, the role of writing is less prominent.

Regarding the interrelation between language and mathematics, text comprehension as well as oral fluency have proven to be key prerequisites and tools for mathematical thinking (e.g. Abedi, 2006; Daroczy, Wolska, Meurers, & Nuerk, 2015; Kintsch & Greeno, 1985; Leiss, Schukajlow, Blum, Messner, & Pekrun, 2010; Nesher & Teubal, 1975; Prediger, 2018; Prediger, Wilhelm, Büchter, Gürsoy, & Benholz, 2018). In contrast, fewer studies focus on the interrelation between writing and mathematics (Miller, Scott, & McTigue, 2016). Nevertheless, these studies indicate that writing activities in mathematics often have a positive effect on learning and achievement in a broad range of mathematical abilities.

The reason for that is that writing is not merely a form of communication, but that it also has a knowledge-constituting function (see Bangert-Drowns, Hurley, & Wilkinson,

DOI: https://doi.org/10.23770/rt1817

2004, for a review). This is considered the *epistemic function* of writing. Cognitive models of writing emphasize how writing can serve as a means for structuring ideas (Hayes & Flower, 1980), elaborating knowledge (Bereiter & Scardamalia, 1987), and generating new knowledge (Galbraith, 1999). Bereiter (1980) defined an intentional use of the knowledge-constituting function of writing as *epistemic writing*. In his framework, epistemic writing is the most proficient form of writing and can only be employed by writing experts. However, more recent studies have shown that epistemic processes that occur during writing are not limited to experts and to intentional, planned writing activities, but that people of all ages and levels of proficiency can learn through writing (Casa et al., 2016; Galbraith, 1999; Pohl & Steinhoff, 2010). To distinguish these broader knowledge-constituting writing activities from expert epistemic writing, they are commonly referred to as *writing-to-learn* (Keys, 1999).

Individual students can engage in writing-to-learn, but it can also be implemented for groups of students working collaboratively. We refer to writing-to-learn activities that are embedded in a collaborative learning setting as *collaborative epistemic writing* (CEW).

Research on writing-to-learn in mathematics mostly focuses on mathematical problem solving through explanatory and informative writing activities, but hardly on other areas of mathematics education like argumentation and proof. From a theoretical perspective, especially CEW could be a highly efficient tool for supporting the acquisition of *mathematical argumentation competence* (MAC). Mathematical argumentation is a highly social and interactive activity that requires communicative skills and the use of proper mathematical language (Vogel et al., 2016). Collaborative learning is a tool well-suited to the promotion of these skills (Kollar et al., 2014). Engaging students in collaborative writing should actively support and structure the use of language even better than oral communication alone. However, previous research has shown that the design of interventions for fostering MAC requires a multitude of considerations to become effective. For CEW, these considerations have not been made explicit in research so far.

2 Goals of this paper

In this paper, we review fundamental characteristics of writing to-learn in mathematics, providing some general guidelines on how writing-to-learn activities can be designed. We will then focus on the more specific activity of CEW and discuss its potential in fostering MAC. This paper is thus guided by the following questions:

(1) What consequences for designing writing-to-learn activities in mathematics education can be derived from existing research and theoretical considerations?

(2) What specific implications can be drawn for designing CEW activities to foster the acquisition of MAC?

To that end, we will first review existing research both on individual writing-to-learn and on CEW to derive practical guidelines for a successful integration in mathematical education. This builds on a review of empirical results as well as theoretical considerations. To answer (2), we make use of previous findings on the acquisition of MAC in undergraduate students to specify a theoretical foundation for fostering MAC through CEW. Based

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on this foundation, we then derive implications for designing CEW activities and for future research.

3 Writing-to-learn in mathematics

3.1 Empirical findings

A number of studies have pointed out a theoretical connection between processes of writing and mathematical thinking. For example, writing processes have been identified to bear a remarkable resemblance to Pólya's steps in mathematical problem solving (Bell & Bell, 1985; Mendez & Taube, 1997; Pólya, 1945). Similarly, Applebee (1984) emphasizes the relation between writing and mathematical reasoning, stating that "good writing and careful thinking go hand in hand" (p. 577). Hence, in mathematics, writing has been identified both as a goal as well as a tool for fostering other mathematical skills (Powell, Hebert, Cohen, Casa, & Firmender, 2017). It is assumed that writing mathematical texts such as explanations, observations, proofs or justifications can help students build a more elaborated and substantial understanding of mathematical contents.

Only few studies analyzing the role of writing in mathematics make use of empirical data. Three literature reviews give an overview over these studies: Of the studies published before 2000 of Bangert-Drowns et al. (2004) and 27 studies referred to mathematics. Miller et al. (2016) reviewed another three studies from the field of mathematics published in the period from 2000 to 2013. With the focus on mathematics, Powell et al. (2017) reviewed 29 studies published between 1990 and 2015, 17 of which included an intervention design. Notably, the studies included in the three reviews hardly overlap. Of the three studies mentioned by Miller et al. (2016), two were also included by Powell et al. (2017), while there is no overlap between Powell (2017) and Bangert-Drowns et al. (2004), which results in a total number of 45 relevant studies in the last 20 years. Compared to reading in mathematics, research on writing-to-learn can thus be considered underrepresented (Miller et al., 2016).

For studies by Bangert-Drowns et al. (2004) including writing activities in mathematics classrooms report a weighted mean effect size of $d = 0.19$ for writing-to-learn interventions compared to control groups. Most of these studies (25) included informational writing tasks, inviting students to describe processes, construct reports or create examples. Only few studies furthermore featured personal writing activities (11), including journals or documenting personal experiences. A metacognitive reflection of the ongoing learning process was prompted in the same eleven studies. Overall, studies reported in the review that included such prompts produced significantly higher effect sizes of $d = 0.26$ compared to studies without prompts for metacognitive reflection ($d = 0.09$).

In an expertise, Casa et al., (2016) sought to clarify the different types and purposes of writing-to-learn in mathematics. They distinguish between *exploratory, informative, argumentative,* and *creative* types of writing. All these activities serve different purposes and goals. The 17 studies included in the review by Powell et al. (2017) were coded according to this distinction. 14 included exploratory writing, while creative writing (2) and argumentative writing (4) were less often implemented. In all seven studies with a control group design, the treatment groups outperformed the control groups. However, effect sizes were not reported by Powell et al. (2017).

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In conclusion, existing reviews indicate that writing-to-learn can, in general, support mathematical learning and thinking. They illustrate that the majority of studies include exploratory or informative writing activities, while argumentative writing was utilized by only a handful of studies. Still, the specific processes behind writing-to-learn that cause positive effects on learning are often not analyzed in detail (Bangert-Drowns et al., 2014).

3.2 Theoretical considerations of writing-to-learn in mathematics

Research has theorized several explanations for the benefits of writing-to-learn for mathematical thinking. We consider the following aspects to be most important:

(a) Written texts are permanent. Therefore, they can easily be reviewed by the writer (Applebee, 1984) or an instructor (Bell & Bell, 1985). This enhances self-regulation (Glogger, Holzäpfel, Schwonke, Nückles, & Renkl, 2009) and reduces cognitive load (McCutchen, 1996; Sweller, 2010). When adequately prompted, the written text can be used to channel students' focus on specific elements of the task or learning process. Writing initially occupies additional working memory capacity, but students benefit from this trade-off at a very early stage of writing proficiency (McCutchen, 1996).

(b) Written texts are explicit. Written mathematical statements do not only have to be valid in their content, but also in their form (e.g. Ottinger, Kollar, & Ufer, 2016). Therefore, ideas need to be constantly verbalized and structured in a negotiation between the *content space* and the *rhetorical space* (Bereiter & Scardamalia, 1987). Thoughts and ideas that might be sufficiently well articulated for oral or mental purposes often need to be specified, elaborated, or condensed before written down. This structuring is therefore effectively stimulated by writing. Furthermore, the explicitness again helps reviewing results and reduces illusions of understanding, because mistakes are not overlooked as easily (Glogger et al., 2009).

(c) Compared to speech, writing is slow and self-paced, which offers writers time to review the written product even during the process of writing (Emig, 1977). Moreover, a slower pace offers possibilities for deeper processing and can prevent superficial errors.

(d) Besides the explicitness, the structure of texts can provide a frame for organizing ideas and their relation (Applebee, 1984). This refers not only to the linguistic structure, but also to the spatial arrangement of the text, including graphical elements as arrows or highlighting. Structuring is an essential part of solving mathematical problems (Kintsch & Greeno, 1985).

(e) Writing refers back to prior knowledge and thereby activates cognitive resources, which is considered a fundamental prerequisite for learning (Bereiter & Scardamalia, 1987).

(f) Writing, especially when guided through prompts, can enhance the use of cognitive and metacognitive strategies (Glogger et al., 2009; Pugalee, 2001). Mathematical thinking benefits from metacognitive and heuristic strategies, because mathematical thinking often requires a flexible, non-algorithmic solution process (Glogger et al., 2009; Kollar et al., 2014; Pólya, 1945).

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3.3 Collaborative epistemic writing

Most studies on writing-to-learn focus on individual expressive and creative writing. They consider these forms of writing to be best suited for individual knowledge constitution. In contrast, Tynjälä, Mason, and Lonka (2001) propose that writing-to-learn is more effective when it is integrated with social interaction. We refer to this form of writing-to-learn as collaborative epistemic writing (CEW). Keys (1999) argues that social writing could facilitate learning even better than individual writing. She points out that since the written product is potentially reviewed or criticized by the learning partners, writers might try to make it less speculative compared to drafts and notes they write for themselves. This should increase the elaboration and revision of written texts (Bereiter & Scardamalia, 1987). Nevertheless, studies adopting CEW are still rare (Cross, 2009; Keys, 1994, 1999; Tynjälä et al., 2001).

There are many fields in mathematics where CEW might be a powerful tool to enhance students' learning outcomes. Most research on writing-to-learn in general and on CEW in particular focuses on problem solving activities. However, other areas of mathematics education have proven to be interwoven with language proficiency and therefore might be especially amendable to writing-to-learn, e.g. word problem solving (Daroczy et al., 2015). Another example is argumentation and proving. While fostering MAC is a key goal of mathematical education research, the possibilities to include writing-to-learn activities have rarely been considered. Starting from what is known about successfully fostering MAC, we outline this possible approach from a theoretical perspective.

4. Mathematical Argumentation Competence

We define MAC as the competence to construct arguments for and against mathematical claims and to generate or inquire mathematical conjectures (e.g. Koedinger, 1999). At university, mathematics is considered a discipline that strongly relies on activities such as argumentation and evaluation of proofs (Healy & Hoyles, 1998; Thurston, 1994). This is also reflected on the school-level in national curricula like the *Common Core Standards* in the United States (CCSSI, 2017) or the German *Bildungsstandards* (KMK, 2015). Similarly, large-scale studies like PISA or TIMSS include argumentation in their frameworks (IEA, 2013; OECD, 2016). Hence, MAC can be cosidered one of the fundmental goals of Mathematics Education.

Our definition of MAC further includes two major components. First, the content specific, individual component refers to the ability to individually generate mathematical arguments, evaluate these arguments and combine them for a valid mathematical proof. Second, the social-discursive component refers to content-general abilities to engage in a discussion, to bring forward own arguments, critique others' arguments with counterarguments, balance different arguments and reach a synthesis of arguments in a social, collaborative situation (Kollar et al., 2014). This distinction illustrates how mathematical argumentation is a highly social and interactive activity that requires communicative skills (Vogel et al., 2016).

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4.1 Collaborative learning processes that foster MAC

Individual learning predominantly adresses the individual, content-specific component of MAC, while collaborative learning has proven to foster both the indiviual and the social-discursive component of MAC (Kollar et al., 2014). The way how collaborative learning can support the acquisition of the social-discursive component of MAC has been subject to a number of studies.

Kollar et al. (2014) fostered prospective teacher students' MAC in an intervention study prior to the start of their university program. During a two-week mathematics course, students regularly engaged in computer-supported collaborative problem solving. During these sessions, groups of two students solved mathematical argumentation tasks, actively engaging in mathematical argumentation. To evaluate the courses, MAC was assessed at the beginning and the end of the course. In the study, all participants were working in pairs and were provided with a graphic tablet for handwritten notes that was connected to the computers. Since participants worked on individual computers, it was necessary to use the tablet to write down and illustrate arguments for the partner. During the sessions, it was observed that these opportunities to write were frequently used by the students, that the notes seemed to play an important role in many collaborative learning processes and were often used as references for the shared solution.

Vogel et al. (2016) further analyzed the written communication between students in this study while they were working collaboratively on a mathematical argumentation task with a specific focus on constructive, dialogic, or dialectic activities (Asterhan & Schwarz; Chi & Wylie, 2014; Wegerif, 2008). They showed that dialectic transactivity (i.e., responding to the learning partner's contribution in an argumentative way by critiquing and/or integrating their learning partner's contributions) is an important factor in fostering MAC.

4.2 The role of guidance in collaborative learning

Kollar et al. (2014) discuss a number of beneficial aspects of collaborative learning for fostering MAC. Specifically, prior research has shown that when collaborative learning is not guided, it is often less effective than individual learning because collaborators do not engage in high-level collaborative processes (Kollar, Fischer, & Slotta, 2007). Thus, Kollar et al. (2014) provided students with *collaboration scripts* and *heuristic worked examples* as means to structure the collaborative learning. Collaboration scripts offer students a clear structure that specify, distribute and sequence learning activities among learners (Kollar, Fischer, & Hesse, 2006). Many studies have shown that collaboration scripts have a large effect on learning social-discursive argumentation skills (see Vogel, Wecker, Kollar, & Fischer, 2017, for a meta-analysis). Heuristic worked examples consist of a problem formulation, solution steps, and the final solution of a problem (Renkl, 2002). Learners follow the solution process of a fictitious peer. In addition to common worked examples, the learner is provided with possible heuristic strategies, e.g. for planning and evaluating mathematical arguments (Reiss & Renkl, 2002; Schworm & Renkl, 2007). The superiority of learning with heuristic worked examples over training corresponding problems without further guidance has been shown in numerous studies (e.g. Hilbert, Renkl, Kessler, & Reiss, 2008; Reiss, Heinze, Renkl, & Groß, 2008; Renkl, Hilbert, & Schworm, 2009).

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The guidance by collaboration scripts and heuristic worked examples should enhance students' individual and collaborative learning processes. Positive effects of the guidance can partially be explained by the *cognitive load theory* (Sweller, 2010). The basic assumption of the cognitive load theory is that the human working memory capacity is limited. From this perspective, providing guidance can reduce the cognitive load needed to structure the collaborative problem-solving process. Thereby, working memory capacity is made available for high-quality processes needed for mathematical argumentation as knowledge construction, elaboration, and reviewing arguments (Schwaighofer et al., 2017). A further explanation for the effects of the guidance is that affordances of both collaboration scripts and heuristic worked examples induce deep cognitive elaborations (e.g. discussions, explanations, critiquing, questioning and answering) that lead to better learning gains (King, 2007). Beyond that, both offer possibilities for metacognitive reflection through prompts which in turn enhance learning as well (Renkl, Hilbert, & Schworm, 2009; Schworm & Renkl, 2007).

Guidance leading to engaging in collaborative learning, reducing cognitive load, and prompting metacognitive strategies has been determined to be a key factor in the successful acquisition of MAC. Similarly, these are also key principles behind the benefits of writing-to-learn. Studies on fostering MAC through collaborative learning therefore also provide a number of arguments for using CEW to foster MAC.

4.3 Previous empirical research on fostering MAC with CEW

To our knowledge, only three empirical studies utilized CEW to foster MAC. Studies by Kosko (2016), and Kosko and Zimmermann (2017) consider argumentation skills and writing but focus on the argumentative writing process as outcome and do not analyze effects on MAC. The study by Cross (2009) is the only one investigating a relation between CEW and mathematical argumentation. In her study, Cross (2009) found that engaging students in writing and argumentation tasks can enhance their learning of the mathematical content, which can be considered the individual component of MAC. Although no interaction effects are reported, she assumes that the two activities could provide synergetic effects based on theoretical assumptions.

Apart from that, there is no empirical research on writing-to-learn in the context of MAC. Moreover, no theoretical foundation exists regarding the question if and how writing-to-learn can influence the learning of MAC.

4.4 Theoretical effects of CEW on MAC

Concluding from the theoretical considerations based on successful studies fostering MAC and previous research on CEW, we propose several reasons why writing should support the acquisition of MAC in a collaborative setting, in addition to the general effects of writing-to-learn.

The process of collaborative mathematical argumentation is believed to consist of three reiterating phases: constructing an argument, answering with an argument, and building a consensus (Kollar et al., 2014; Leitão, 2000). We postulate that each of these phases is distinctively supported by activities of CEW.

4.4.1 Constructing an argument. Collaborative writing is a social activity. Similarly, mathematics and MAC in particular are considered inherently social and constructive

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activities (Cross, 2009; Kollar et al., 2014). Therefore, we assume that CEW encourages students in social interaction and initiates the motivation to engage in argumentation and to construct an argument to be communicated to a learning partner, especially when guidance is provided. The potential review by the learning partner should further increase efforts to construct a valid and concise argument (Casa et al., 2016; Keys, 1999). When the writing process is prompted or structured, it can support the initial formulation of an individual argument.

4.4.2 Answering with an argument. Through collaborative monitoring of the work of the other, learners can constantly cross-evaluate their work (Bell & Bell, 1985; Cross, 2009). This is necessary for formulating arguments supporting or refuting the partner's argument and for the second phase of argumentation. In addition, the role of specific feedback on the validity and conformity of an argument is especially important for learning MAC since the acceptance of mathematical arguments is often subject to social norms (Reiss & Ufer, 2009). Through prompts, metacognitive reflection of the process can be initiated.

4.4.3 Building a consensus. Finally, collaboratively written texts can constitute shared knowledge (Cross, 2009). This shared knowledge ideally reflects an explicit consensus between the two learners. Furthermore, the permanent fixation of steps in the argumentation reduces the individual cognitive load (Sweller, 2010). By enabling learners to fixate information and to channel their cognitive load towards the common argumentation process, writing could optimize collaborative mathematical argumentation (McCutchen, 1996). In general, mathematical argumentation is a very resource-intensive activity since students need to engage on the level of content, the level of argumentation, and the strategic level in parallel (Schworm & Renkl, 2007). Ultimately, MAC should include the ability to eventually find a proof or refutation of a mathematical conjecture in a written form (Kollar et al., 2014). Writing down intermediate steps and making use of a given structure (e.g. Boero, 1999) may help to organize and structure the final proof (Kollar et al., 2014).

Overall, these considerations strongly support our assumption that CEW as a writing-to-learn activity in a collaborative learning setting could foster MAC from a theoretical perspective. A prerequisite is a thorough implementation and design of the CEW activities, particularly the guidance that students receive. Following these first conclusions, we will discuss two examples for the use of guidance in CEW: collaboration scripts and heuristic worked examples.

4.5 Designing CEW activities: guidance in writing-to-learn

The way how writing-to-learn is instructed is important for the quality of the written product and hence for the positive impact on learning (Arnold et al., 2017; Cross, 2009; Powell et al., 2017). Guidance in the form of instruction or prompts for metacognitive reflection can support writing interventions (Bangert-Drowns et al., 2004; Glogger et al., 2009; Nückles, Hübner, Dümer, & Renkl, 2010). To decrease cognitive load, prompts need to be specific, since otherwise they provide additional cognitive challenges (Glogger et al., 2009). When prompts are presented but are not necessary for the learner, they can also have detrimental effects. Accordingly, prompts should be faded when learners reach a certain proficiency (Nückles et al., 2010).

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Heuristic worked examples offer a solution of a fictitious peer. On the one hand, this can foster students' ability to review and follow mathematical argumentation. At the same time, cognitive resources needed for solving the argumentation task are reduced. Having a worked example could motivate CEW, but vital parts of CEW could be impeded. By providing a solution, the need for individual text production is reduced. In particular, the social-constructive aspect of CEW and the construction of shared knowledge might be decreased by heuristic worked examples compared to problem solving.

In contrast, collaboration scripts actively prompt students to raise arguments, to evaluate each other's arguments, and to come to the synthesis of their arguments that constitutes their shared knowledge. However, these prompts are not specific to writing, but aimed at the collaborative argumentation. For example, learners are prompted to review the partner's argument, but not to review the partner's written product or give a written statement. In addition, guidance can offer a structure to organize the argumentation process. This might be overwhelming for students with lower working memory capacity (Kollar et al., 2014; Schwaighofer et al., 2017).

Both ways of guidance show potential to actively promote CEW. At the same time, they need to be carefully designed for learners to benefit from the additional workload.

4.6 Designing collaborative epistemic writing interventions to foster MAC

Based on our considerations, we conclude by proposing guidelines for designing CEW activities aiming to foster MAC. We adopt five conditions for successful integration of writing as a learning tool from Tynjäla et al. (2001), which we extend to the specific case of fostering MAC though CEW activities.

1) Writing should promote active knowledge acquisition, rather than reproduction. When guidance is provided, it has to be assured that students are still actively engaged in the learning process. Even though revising is a key activity during CEW, this revision needs to exceed a correction of mistakes. In developing MAC, this might be achieved by encouraging learners to formulate brief written statements or to reformulate a partner's argument, rather than simply marking errors.

2) Writing should account for and make use of students' prior knowledge. This is especially important in CEW to assure that learners both contribute their ideas, which is a prerequisite for constructing shared knowledge. This might be achieved by including phases of individual, free writing tasks at the beginning of the CEW. This could include formulating an own hypothesis or collecting and structuring first ideas for arguments at the beginning of collaborative learning sessions.

3) The writing task should encourage learners to reflect their experiences. This means that students should regularly be encouraged to review the learning process on a meta-level, independent of the content. This is especially important for MAC. Evaluation and revision are key activities in MAC. Furthermore, mathematical argumentation typically includes at least two levels of learning processes: Learning about the mathematical content and learning about good mathematical argumentation. Therefore, encouraging learners to engage in reflecting both levels of their work seems necessary for successful CEW.

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4) The writing task should enable learners to actively apply their knowledge. This means that the writing task should not be merely a documentation of final results but should be part of the learning process. For MAC, it is important to make use of the full potential of writing-to-learn, for example by encouraging learners to find examples, to simply try new ways of argumentation and to use writing as a tool for mathematical thinking.

5) Writing tasks should be integrated with other activities such as small-group discussions or reading. In CEW, the social aspect of this condition should be met in most occasions, but the integration of individual work phases should not be neglected. This can be accomplished by including material and prompts that specifically encourage students to engage in such activities, for example discussing initial hypotheses, read alternative argumentation or exchange ideas with others.

4.7 Analyzing CEW: how text quality matters

Finally, an evaluation of CEW activities requires a valid measure of their quality. Reviews on writing-to-learn concur that effects are typically not large and differ between writing activities (Applebee, 1984; Bangert-Drowns et al., 2004; Miller et al., 2016; Powell et al., 2017). One critical finding is that the quality of the written text is a better predictor for a successful intervention than the kind of writing instruction (Arnold et al., 2017; Kosko, 2016) or the time spent writing (Bangert-Drowns et al., 2004). For example, Arnold et al. (2017) found that engaging students in free recall (asking participants to recall everything they remember after reading a text) and essay writing did not differ in their effect on learning, but the quality of the written text did predict achievement in a subsequent test. Consequently, a majority of studies on writing-to-learn consider the quality of student writing samples as a possible outcome or mediator in writing-to-learn interventions (Bangert-Drowns et al., 2004; Miller et al., 2016).

Furthermore, the epistemic function of writing implies a need to analyze the process of writing, not merely the written product (Tynjälä et al., 2001). Nevertheless, the *process* of writing is still rarely analyzed (Bangert-Drowns et al., 2004; Glogger, Schwonke, Holzäpfel, Nückles, & Renkl, 2012; Miller et al., 2016). This is surprising, since writing is considered a dynamic cognitive process including planning, translating, and reviewing (Hayes & Flower, 1980) that might not be adequately evaluated by focusing on the final product. We assume that analyzing writing-to-learn should always include an observation of the writing process. This is especially relevant for fostering MAC through CEW, since mathematical argumentation in a social context is a process, rather than a product. For observing writing processes, research can make use of new technologies, for instance *Smartpens* that allow a timestamped parallel recording of writing and audio (Schwaighofer et al., 2017).

5. Conclusion

Writing-to-learn is a promising yet comparatively uncharted field of mathematics education. Processes that could foster mathematical thinking and learning are highly plausible but have not been systematically described. Similarly, benefits for specific fields of mathematics like argumentation and proof lack a contemporary theoretical foundation.

In this study, we outlined key benefits of writing-to-learn in general and of CEW in particular. Combining some of the fundamental aspects of writing-to-learn with experiences from fostering MAC, we formulated processes that we believe show how CEW can explicitly support MAC.

Our starting point was the question if CEW could, from a theoretical perspective, foster MAC. Based on our considerations, we conclude that CEW offers plenty of reasons to qualify for such benefits. Yet, CEW activities need to be implemented in a carefully designed learning environment including guidance and prompts, taking into account students' cognitive resources and prior knowledge. We discussed heuristic worked examples and collaborations scripts as two examples for guidance, though other forms of guidance could be similarly suitable.

We ended by proposing practical implications what researchers and teachers should consider in designing CEW activities and how research can analyze the effectiveness of CEW by considering text quality and the writing process. We assume that this can be a foundation for future empirical research and a first step towards a deeper understanding of the possibilities and challenges in mathematical writing-to-learn.

Finally, we believe that the rather specific considerations from this paper can be applied to other domains and learning activities. Writing-to-learn offers possible benefits not only for mathematics, but for subject-matter education in many fields, for example science (Bangert-Drowns et al., 2004). In this paper, we considered fostering MAC in undergraduate students, but we believe that students with a certain level of writing abilities can benefit from writing-to-learn in all age groups. Moreover, processes that go beyond traditional writing activities could arguably still be situated within the framework of writing-to-learn, for example text production on tablet computers, or creative processes involving audio or video material. We regard this generalizability as one of the great potentials of writing-to-learn in contemporary subject-matter educational research.

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DOI: https://doi.org/10.23770/rt1817